Gainford C E Primary School

PROGRESSION

IN

WRITTEN

CALCULATION

CONTENTS

PROGRESSION IN CALCULATION

From Foundation Stage to Year 6

The aim for written calculations is different from the aim for mental calculations. With mental work, the aim is to teach children a repertoire of strategies from which to select. With written calculations the ultimate aim is proficiency in a compact method for each operation.

Mental calculation

Strategies for mental calculation are introduced from Y1 or Y2 to Y3 and developed further in Y4, Y5 and Y6. All children, apart from those with significant special educational needs, should be introduced to the full range of mental calculation strategies when they have the necessary pre-requisite skills. Children with significant special needs should learn a narrow range of strategies which are generally applicable.

Written calculations

Building on the mental strategies they have used so that they can understand the process, children need first to be taught to record their methods in an expanded form. When ready, they are taught how to refine the recording to make it more compact.

The methods and layouts to be taught for each operation are detailed in this document.

Challenges to teachers

- Ensuring that recall skills are established first so children can concentrate on a written method without reverting to first principles
- Making sure that, once written methods are introduced, children continue to look out for and recognise the special cases that can be done mentally;
- Catering for children who progress at different rates; some may grasp a compact method of calculation while others may never do so without considerable help;
- Catering for children who can carry out some standard methods successfully, e.g. for +, but not - ;
- Recognising that children tend to forget a standard method if they have no understanding of what they are doing

Often the compactness of a vertical method conceals how mathematical principles are applied, e.g. children may use place value when working mentally, but be confused in written work because they do not understand how place value relates to 'carrying'. There can be long-lasting problems for those taught compact, vertical methods before they understand what they are doing.

Simply correcting children's errors may help in the short-term, but not permanently. They need to understand why a particular method works rather than simple following a set of rules. They can then fall back to a simpler method if uncertain or to check their answer.

NOTE: In the following guidance, suggestions are given as to when written methods and particular layouts should be introduced. However, the most importance thing to consider rather than children's age, is whether they have the necessary pre-requisite skills.

Progression towards a written method for addition

Before the introduction of formal written methods for addition, children should be able to:

1. recall all addition pairs to 9 + 9 and complements in 10;
2. add mentally a series of one-digit numbers, such as 5 + 8 + 4;
3. add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;
4. partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways.

YEAR 2

Mental method using partitioning:

- without bridging tens, e.g. 34 + 23
- bridging tens, e.g. 37 + 24
- bridging hundred, e.g. 47 + 76 (L3)

e.g. 4 7 + 7 6

4 0 + 7 0 = 1 1 0

7 + 6 = 1 3

1 1 0 + 1 3 = 1 2 3

YEAR 3/ YEAR 4 (If children are not ready for this method in Y3 refer to Y2 methods)

Build up starting with 2 digits + 2 digits in Y3

Introducing the compact layout

- no carrying (demonstration stage only), e.g. 54 + 35, 326 + 271
- carrying from units to tens, e.g. 47 + 26, 368 + 423
- carrying from tens to hundreds, e.g. 368 + 481
- carrying from units to tens and tens to hundreds, e.g. 47 + 76, 368 + 478
- a mixture of 'carries'

E.g. demonstrate the method children currently use alongside the new method. Children should then practise the new method. (3 6 8 + 4 7 8)

```
   H  T  U
   3  6  8
+ 4  7  8
  ¹  ¹
  8  4  6
```

When the compact layout is introduced, it is helpful to use the place value headings of HTU.

When the compact layout is introduced, the language of place value should continue to be used but when children are confident, they will use 'digit-speak', e.g. for the addition of two three-digit numbers above, they are likely to say:

- 8 add 8 is sixteen; 6 in the answer and carry 1
- 6 add 7 is 13 plus the carry 1 is 14; 4 in the answer and carry 1
- 3 add 4 is 7 plus the carry 1 is 8
- Answer is 846

This is a form of shorthand that speeds up the process of addition. If children consistently carry out a range of calculations correctly, it is likely that they understand the process. However, teachers should occasionally check their understanding by asking children to explain what exactly they mean at each stage of the calculation, e.g. what does that 'carry 1' really mean?

NOTE: Children will meet fairly simple additions of £.p during Y4. Some will be able to use the compact layout but those who have not learnt how to use this layout should change pounds to pence and add using a compact layout.

YEAR 5 and YEAR 6

Extending the compact layout

- More than two numbers to be added
- Different numbers of digits
- Decimals (same number of decimal places), money and measures
- Decimals (different number of decimal places)

Progression towards a written method for subtraction

Counting up

Counting up, used as a written method for subtraction, makes use of the number line to keep track of the calculation:

- to find the difference
- to compare two quantities, values or measurements
- to subtract

YEAR 2

1. Children should be able to partition 2-digit numbers into tens and ones

   TU – multiple of 10, e.g. 76 – 40

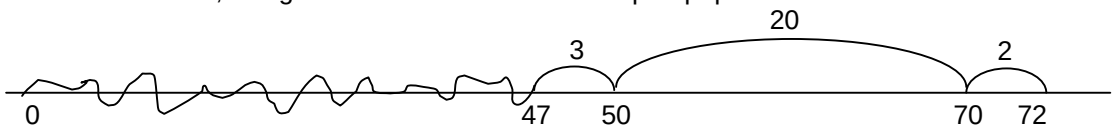
2. Children should be able to recognise the multiple of 10 following the smaller number.

   Multiple of 10 – TU, e.g. 80 – 53

3. Children should be able to use an empty number line and bridge through multiples of 10

   TU – TU, e.g. 72 – 47

   If children are confused by having to add to solve a subtraction calculation, use the approach described below, using either a number line or a strip of paper.



   Label the number line 0 to 72.

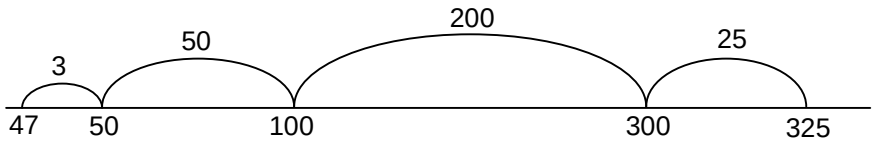
   - Ask a child to position the number to be subtracted on the number line.
   - Explain that the first section of the number line is worth 47, which is to be subtracted or taken away. Then cut off that section if using a strip of paper, rub it out or scribble it out if using a whiteboard.
   - Explain that the rest of the line is what is left and, to find the answer, children must find what it is worth by counting. Counting may be done in several ways, but encourage counting up, bridging through multiples of 10, as this principle is developed for more difficult calculations.

YEAR 3

Continue with 2 digits - 2 digits until children are ready for:

4. Children should be able to bridge through multiples of 100. They should be taught to reduce the number of steps by applying their knowledge of bonds to 100.

   HTU – TU, e.g. 123 – 86, 325 – 47



   Children add mentally to find the answer, making jottings as necessary.

250 + 25 + 3 = 278 325 – 47 = 278

Reducing the number of steps – children need to be confident with bonds to 100 and be able to internalise the addition from 100 to 300 and then to 325 as one step.

225 + 53 = 278 325 – 47 = 278

5. HTU – HTU, e.g. 763 – 278

6. £ - £.p, e.g. £10 - £5.87

7. £.p - £.p, e.g. £15.48 - £3.75

Children should be confident in carrying out subtraction by counting up to the levels described above before being introduced to the formal method of decomposition. Some children find decomposition difficult. General advice is that those who will struggle to reach L4 at the end of KS2 should continue to use the counting up method.

Decomposition

Before the introduction of the formal written method for subtraction (decomposition), children should be able to:

1. recall all subtraction facts to 20;
2. subtract multiples of 10 (such as 160 – 70) using the related subtraction fact,16 – 7, and their knowledge of place value;
3. partition two-digit and three-digit numbers into multiples of one hundred, ten and one in different ways (e.g. partition 74 into 70 + 4 or 60 + 14).

YEAR 4

<u>Expanded layout – MODEL ONLY</u>

<u>Introduce compact method as quickly as possible</u>

- TU – TU, then HTU – HTU, no exchanges, e.g. 68 – 44, 254 – 123. Start with least significant digits first. Demonstration phase only.
- TU – TU, then HTU – TU and HTU – HTU, exchange from tens to units, e.g. 62 – 44, 173 – 38, 774 - 248
- HTU – HTU, exchange from hundreds to tens, e.g. 553 - 272
- HTU – HTU, exchange from tens to units and from hundreds to tens, e.g. 635 – 278
- Demonstrate the compact method alongside the expanded layout.

  e.g. 635 – 278

500 [1]20
~~600~~ ~~30~~ [1]5
\- 200 70 8
300 50 7 = 3 5 7

H T U
5 [1]2
~~6~~ ~~3~~ [1]5
\- 2 7 8
3 5 7

> With the compact layout, children will use 'digit speak', i.e. 5 subtract 8. Can't so need to use a ten. That leaves 2 (tens). Therefore it is helpful to use the place value headings of HTU.

YEAR 5 /6

<u>Continuing with the compact layout</u>

- Children use the compact method. If they continue to make errors after some practice, return to the compact method, or consider counting up.

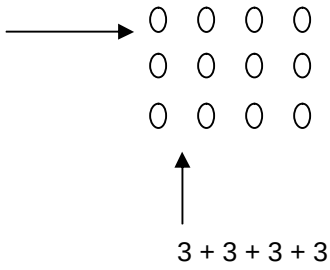
<u>Extending the compact layout</u>

- Subtract numbers with up to four digits, including numbers with different numbers of digits
- Subtraction with numbers involving zeros e.g 5001 - 2345
- Subtract decimals, money and measures

Progression towards a written method for multiplication

YEAR 2

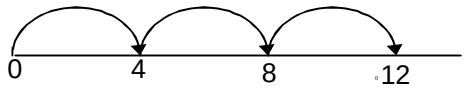
- Pictorial recording as children develop understanding of repeated addition, e.g. counting in sets of two, e.g. sets of two pence coins, five, e.g. tally bundles, tens, e.g. sets of 10 pennies..
- Use of arrays to illustrate repeated addition.



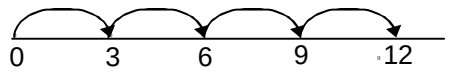
4 + 4 + 4 = 12

3 + 3 + 3 + 3 = 12

- Record repeated addition on a number line
  e.g. for the array above,



4 + 4 + 4 = 12 leading to four, three times or 4 x 3



3 + 3 + 3 + 3 = 12 leading to three, four times or 3 x 4

Using such models will help develop children's understanding of the commutativity of multiplication,
e.g. 6 x 2 = 2 x 6

Before the introduction of formal written methods for multiplication, children should be able to:

- recall multiplication facts for the tables used;
- partition numbers into multiples of one hundred, ten and one;
- work out products such as 70 × 5, 70 × 50, 700 × 5 or 700 × 50 using the related fact 7 × 5 and their knowledge of place value;
- add two or more single-digit numbers mentally;
- add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;
- add combinations of whole numbers using the column method.

YEAR 3/ YEAR 4

Year 3 use the grid method with tables children are confident with

The grid layout

Lower achieving children will continue to use this layout even when compact methods are introduced to higher achievers.

- TU x U, e.g. 37 x 6. Demonstrate alongside mental method. Children use the grid layout.

3 0 x 6 = 1 8 0
7 x 6 = 4 2
1 8 0 + 4 2 = 2 2 2

x	6
30	180
7	42
	222

This layout makes addition of the partial products much easier.

Short multiplication

- TU x U, e.g. 3 7 x 6 Demonstrate alongside the grid layout. Children use short multiplication.

x	6
30	180
7	42
	222

H T U
3 7
X 6
2 2 2

- HTU x U, e.g. 4 7 6 x 7
- £.p x U, e.g. £3. 64. x 8

Note: Children who are continuing to use the grid layout should, when working with money or measures, convert to the smaller measure e.g. convert £.p to pounds, and then back to the original units if required.

YEAR 5

Progression to long multiplication

- Grid layout – teens number x teens number, e.g. 18 x 14, TU x teens number, e.g. 56 x 17, TU x TU, e.g. 56 x 27, HTU x TU, e.g. 375 x 83

Following on from the layout used for TU x U, the first number would be partitioned down the left-hand side. However, if children's understanding of the commutativity of multiplication is secure, they will see that the second number could equally well be partitioned at the side.

	20	7	
50	1000	350	
6	120	42	
	1120	392	1512

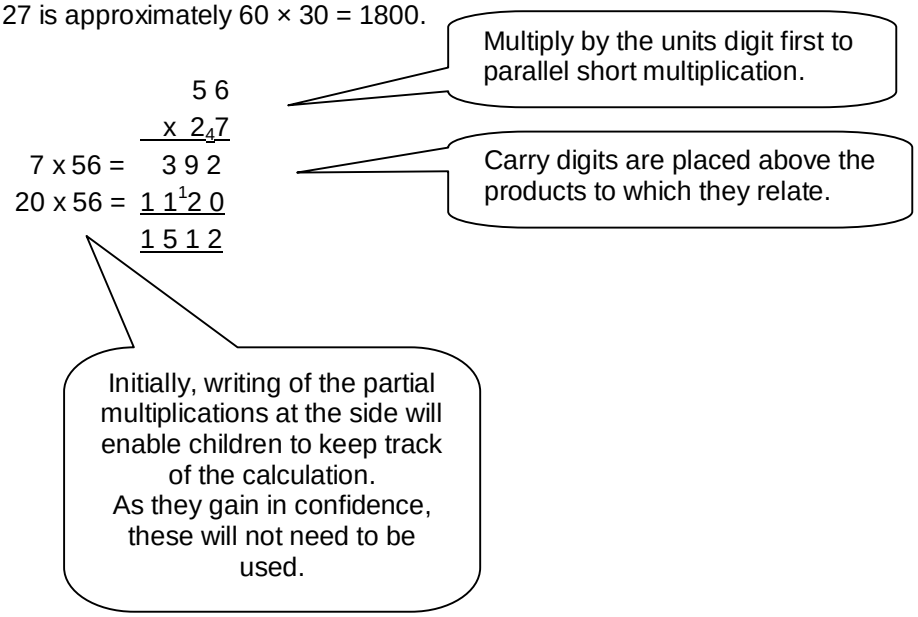
Most children will continue to develop their use of the grid method. Those who are on track to achieve high L4 or L5 at the end of Y6, and are confident with the grid method, should be introduced to the following stages.

YEAR 6

- Compact method for long multiplication - TU x TU, e.g. 56 x 27, HTU x TU, e.g. 274 x 78 (L5)

  Reduce the recording further, making links to short multiplication.

56 x 27
Estimate first.
56 × 27 is approximately 60 × 30 = 1800.

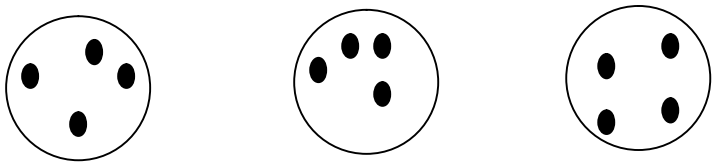


Progression towards a written method for division

YEAR 2

- Use **sharing** to answer division questions such as:

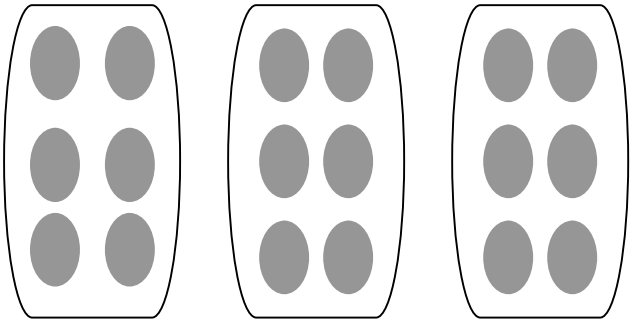
  Share this bag of 12 counters into 3 pots.



- Experience **division as grouping**, such as:

  24 eggs are packed in boxes of 6. How many boxes are needed?

  Take 6 eggs and pack the first box. Continue until there are no eggs left. Then count how many boxes have been used, recording pictorially.



……. etc. until all 24 eggs are packed in boxes.

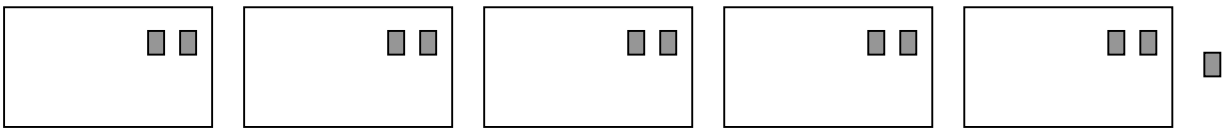
- Experience divisions that give rise to **remainders**, such as, in a sharing context:

  Three friends share 17 marbles equally. How many marbles does each friend get? How many marbles are left over?

  They get 5 marbles each and there are 2 left over.

  ..and in the context of grouping:

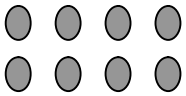
  Grandma is sending Christmas cards. She needs 2 stamps to stick on each envelope. She has 11 stamps, how many cards can she send?



  She can send 5 cards and has 1 stamp left over.

- Relate division to multiplication using arrays.

How many sets of four in 8? →



How many sets of two in 8?

Y3 to Y6

To progress towards short division, children need to be able to:

- understand and use the vocabulary of division;
- partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways;
- recall multiplication and division facts for the tables used;
- recognise multiples of one-digit numbers and divide multiples of 10 or 100 by a single-digit number using their knowledge of division facts and place value;
- know how to find a remainder working mentally, e.g. find the remainder when 48 is divided by 5;
- understand and use multiplication and division as inverse operations.

YEAR 3 / YEAR 4

Expanded layout for short division

Stage 1

TU ÷ U, no remainder and no carrying, e.g. 69 ÷ 3

```
   20   3
  ------
3 | 60   9
```

69 ÷ 3 = 23

Stage 2

TU ÷ U, remainder but no carrying, e.g. 67 ÷ 3

```
   20   2 R 1
  ------
3 | 60   7
```

67 ÷ 3 = 22 R 1

92 ÷ 4 = 23

Stage 3 (compact division)

TU ÷ U, no remainder and no carrying, e.g. 93 ÷ 3

```
    T  U
    3  1
  ------
3 | 9  3
```

There are exactly 30 threes in 90 and 1 three in 3. The answer is 31.

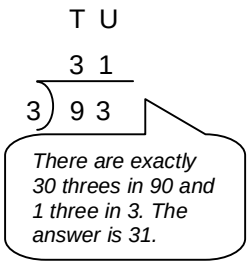
YEAR 4 / 5 (or when children are ready for compact method)

Compact layout for short division

Initially, children will continue to use correct place value vocabulary, but as they become more confident, they can be introduced to 'digit-speak'. Dialogue with place value vocabulary is illustrated for Stages 1-4 and 'digit-speak' shown for Stage 4.

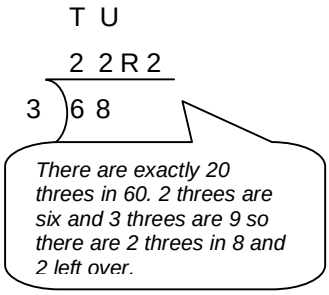
Stage 1

TU ÷ U, no remainder and no carrying, e.g. 93 ÷ 3



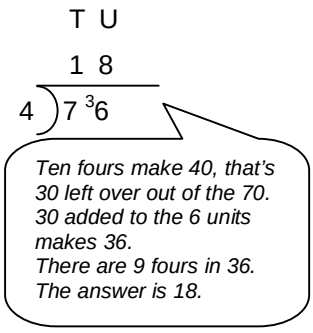
Stage 2

TU ÷ U, remainder but no carrying, e.g. 68 ÷ 3



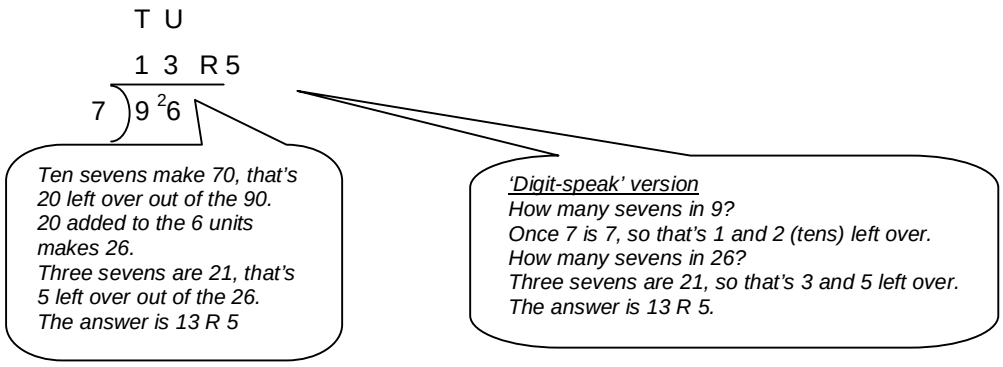
Stage 3

TU ÷ U, carrying from T to U but no remainder, e.g. 76 ÷ 4. When dealing with carrying figures, relate to knowledge of place value.



Stage 4

TU ÷ U with carrying and remainder, e.g. 96 ÷ 7



Extending the compact layout for short division

Order of difficulty of calculations:

1. No remainder, no carrying, e.g. 844 ÷ 4
2. Remainder, no carrying, e.g. 486 ÷ 4
3. No remainder, carrying from T to U, e.g. 860 ÷ 4
4. No remainder, carrying from H to T, e.g. 928 ÷ 4
5. No remainder, carrying from H to T and T to U, e.g. 984 ÷ 4
6. Remainder and carrying, e.g. 743 ÷ 4
7. Examples where consideration needs to be given to the placing of the quotient,

   e.g. 387 ÷ 4

   0 9 6 r 3
   4) ~~3~~ 8 7

   Initially pupils should cross out the hundreds digit and carry it over to the tens as well as placing zero in the quotient.

8. Examples where there are zeros in the quotient, e.g. 818 ÷ 4, 5609 ÷ 8

   2 0 4 r 2
   4) 8 ~~1~~ 8

   0 7 0 1 r 1
   8) ~~5~~ 6 0 9

   Emphasise zero as place holder.
9. Express remainders as fractions, e.g. 387 ÷ 4

   0 9 6 ¾
   4) ~~3~~ 8 7

10. Decimals
11. Measures
12. Divide amounts of money,

    e.g. Grandma emptied her money box. There was £12.46. She shared it equally between her five grandchildren. How much did they each get and how much was left over?

    0 2 · 4 9 r 1

$5\overline{)\cancel{1}^{1}2\cdot^{2}4^{4}6}$

Each child got £2.49 and there was 1p left over.

13. Varied contexts where the remainder is expressed as a whole number, or a fraction, or a decimal or where the quotient needs to be rounded up (e.g. 72 children are going camping. Each tent holds 5 children. How many tents do they need?), or down (The farmer has collected 91 eggs. Egg boxes each hold 6 eggs. How many boxes can he fill?).

YEAR 6

Long division

- List tables facts to solve long division problems

504 ÷ 21 = 24

List tables facts

$$\begin{array}{r} 0\ 2\ 4 \\ \hline 21\overline{)\ 5\ ^{5}0\ ^{8}4} \end{array}$$

21
42
63
84
105
126

- Challenge with extended digits, decimal numbers, money and measures including remainders